# Recharge Response Functions

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## Abstract

**This paper explores the establishment of transfer functions for describing the annual oscillation of unconfined aquifer water levels in response to effective precipitation. A simple saturated zone representation is developed to accompany the unsaturated zone mechanism. Practical examples are drawn from a sample of sites from the chalk and the Permo-Triassic sandstones of England and Wales. Modelled water levels are in many cases good. The technique is most appropriate within the usual range of fluctuation of aquifer water level, with no great change in influence of abstractions, and when it is acceptable to approximate the complexity of unsaturated zone processes in practical analysis.**

## Introduction

This paper addresses the timing of the change in saturated zone water level of unconfined aquifers in response to effective rainfall input. It investigates merits and demerits of the use of a simple approach to approximate unsaturated zone behaviour in its role in determining water table levels over series of annual cycles. Such an approach is most relevant in cases when it is appropriate to simplify the complex hydrology of the unsaturated zone for practical consideration.

Attention is focused on the derivation and performance of transfer functions characterising the timing of changes in unconfined aquifer water levels in response to changes in effective precipitation. As such, the method also involves assumptions and/or approximations concerning the processes active above and below the unsaturated zone, namely evapotranspiration and soil behaviour on the one hand, and saturated zone flow on the other. A standard procedure is used for the former; a simple representation is developed for the latter.

Modelling unsaturated zone transfer as a distributed lag is unspecific as to whether the actual water and/or its pulse is described. This need not be problematical in a number of types of application: it could, though, clearly be a disadvantage with respect to the consideration of water quality. [Quality transfer functions can of course be explored in their own right.]

Attention is directed towards examples from the two main UK aquifers, the chalk and the Permo-Triassic sandstones.

## Method

### Effective precipitation

The effective precipitation used in this paper is that calculated on a monthly basis by the MORECS method (Thompson *et al.*, 1981) as 'hydrologically effective precipitation' for 40 × 40 km grid squares. The reason for its use is its availability over the whole of the Great Britain for the period since 1961 and its general acceptability. This is not to disregard drawbacks to the system which have been noted (see, for example, Harding, 1993) and, indeed, updating in the production of MORECS 2.0 (Hough *et al.*, in press). The 'hydrologically effective precipitation' term of MORECS can include any lateral transfer as well as (approximately) vertical recharge. By concentrating in this paper on unconfined aquifer locations, it is assumed that all hydrologically effective precipitation is ultimately recharge to the regional saturated zone without excessive lateral displacement. The use of the method should be restricted to areas where lateral water transfer is insignificant compared with (approximately) vertical recharge. Thompson *et al.* (*op. cit.*) provide the details of MORECS calculation methods: in short, a modified Penman-Monteith evapotranspiration formulation is used for different land uses and three classes of dual-reservoir soils. Grass estimates for medium texture soil were used here.

*Unsaturated zone transfer*

The essence of the modelling in this paper is to see whether an unsaturated zone unit response function, $u$, can serve to describe unsaturated zone behaviour, albeit in a summarised way, giving

$$R(t) = \int_0^t I(\tau)u(t-\tau)d\tau$$

where $R(t)$ is the recharge reaching the saturated zone over time and $I(\tau)$ is the effective precipitation time series. Does such a convolution integral formulation have a role to play in reproducing aquifer water level time series and could it serve as a potential method of extension of the series over time when a rainfall series is known or assumed and an extension of water levels is sought?

This part of the modelling is linear: this is by no means, however, to say that the rainfall/recharge relationship is so, because of the considerable non-linearities in the rainfall/effective rainfall relationship.

In the work described in this paper the transfer function is used in discrete form, working on a monthly basis. The total recharge to the saturated zone in month $n$ can be written as

$$R_n = \sum_{i=1}^{m} I_{n-i+1}u_i$$

where $u_1$, $u_2$ ... $u_m$ are monthly increments of recharge to the saturated zone resulting from unit effective precipitation, with $u_1$ occurring in the same month as $I_1$. The value of m is determined in the calibration process. The monthly discretisation is well able to pick up annual variations for water resource applications and is appropriate for the type of methodology involved. The function u is derived by trial and error, comparing time series of observed and predicted water levels. This continuous time basis avoids the problem of definition of separate events and of individual conditions of starting water levels. Here, input is a given and variation of the transfer function is explored in terms of resultant water level at a borehole location.

## SATURATED ZONE BEHAVIOUR

Borehole water level data from the National Water Archive records were used for ten unconfined chalk and Permo-Triassic sandstone sites (Fig. 1, table 1) spread over England and Wales. The frequency of water level readings and length of record differ. All records were converted, usually from more frequent readings, to a series of monthly water levels occurring in the first week of the month, allowing up to two months of linear interpolation; otherwise missing values were recorded. This water level was compared with the result of processes modelled up to the end of the previous month. As long a record as posssible was used, between the 1961 beginning of MORECS data and 1991 when Institute of Hydrology holdings of hydrologically effective precipitation stop.

A simple and approximate method for saturated zone behaviour was developed to summarise background conditions, without recourse to groundwater catchment modelling. The aim was to use a simple physical method not relying on full regional modelling but, rather, with the potential to serve such studies. Possible driving variables for determining net saturated zone outflow from a borehole location are effective precipitation and water level. In this paper emphasis was placed on the use of effective precipitation, with a view to the use of the procedure in predictive mode when variations in water levels are likely to be unknown. It was preferred that the functions were established in the same manner as it was envisaged that they would be used: the knowledge of observed water levels is therefore used as a means of calibration and is not an integral part of the procedure.

Over the whole period of record, in the absence of any very great change in initial and final water levels (at a particular time of year), recharge to a particular location on the water table was assumed to approximate the saturated zone net outflow away from that location because of little change in storage, as reflected in the water level values. Ignoring any recharge lag, and any difference in unsaturated zone storage at the two times, the total net outflow over the modelled period was taken to approximate total effective precipitation over the period. The average monthly saturated zone outflow was weighted, on a monthly basis, by the ratio of the previous twelve months' rainfall in relation to average annual rainfall in order to give an estimate of monthly series of net saturated zone outflow at an aquifer location. The rationale for this is that the wetter the previous period, the more likely is a greater rate of outflow: there is not, however, the information to do other than assume a form for this relationship. If the end-of-record water level is very different from initial water level, it is possible to apply an overall gain or loss in deriving the outflow term, or one weighted towards times of changes in abstraction or inputs: this was not invoked in these examples (although site 8, see below, is arguably a candidate).

Recharge from effective precipitation, lagged in a distributed manner by the trial response function, serves to increment the water level at the location, and the local saturated zone net outflow subtracts from it. Values of aquifer specific yield necessary for the conversion of changes in water storage to expression in terms of elevation of water table are derived by calibration. Any differences in specific yield over the zone of water level fluctuation, although they can in principle be included, are here approximated by a constant value, in keeping with the general level of physical detail of the method. Because of the great emphasis on a single water level

*Table 1.* Details of borehole sites and attributes.

| Site | Grid reference | Aquifer* | MORECS Square | Thickness of unsaturated zone† (m) |
|---|---|---|---|---|
| 1 Washpit Farm, Rougham‡ | TF 81381960 | Upper Chalk | 130 | 37 |
| 2 Therfield Rectory‡ | TL 33303720 | Upper Chalk Middle Chalk | 151 | 73 |
| 3 Sandview Cottage, Rockley, Ogbourne St Andrew‡ | SU 16557174 | Middle Chalk ?Lower Chalk | 158 | 12 |
| 4 West Woodyates Manor, Pentridge‡ | SU 0160 1960 | Upper Chalk | 181 | 25 |
| 5 Compton House‡ | SU 77551490 | Upper Chalk ?Middle Chalk | 183 | 39 |
| 6 Furness Abbey | SD 21727171 | Permo-Triassic sandstones unclassified | 90 | 9 |
| 7 Llanfair Dyffryn Clwyd, Ruthin‡ | SJ 13745556 | Permo-Triassic sandstones unclassified | 113 | 2 |
| 8 Weeford Flats‡ | SK 14400464 | Keuper Sandstones | 125 | 6 |
| 9 Stores Cottage, Huntley | SO 71701970 | Permo-Triassic sandstones unclassified | 147 | 3 |
| 10 Bussels Farm, Huxham‡ | SX 95289872 | Permo-Triassic sandstones unclassified | 178 | 6 |

* From British Geological Survey well records
† Approximate mean value over modelled period
‡ National Water Archive index well

observation, of unknown quality, as an initial reference level for the whole modelled water level series, the option of using any of the first three start-of-year levels was allowed to give the least overall error. It is numerically necessary to allow a run-in period at the beginning of the convolution of rainfall series and response function of at least the length of the maximum recharge lag.

It is arguable whether to use split-record calibration and validation periods or to use all available records to establish the transfer function parameters. The latter approach was chosen with the aim of deriving as good a set of parameters as the data allowed. [It is possible to gauge the outcome of a split-record approach by rough comparisons of goodness-of-fit over different parts of the record.]

## Results

Figures 2 and 3 show observed and modelled aquifer water levels according to the method described above; the optimised parameter values are shown in the figures. Because this method was being developed, as opposed to being an established technique, no goodness-of-fit criteria were specified in advance for acceptance. The general quality of the results is encouraging in pursuing this approach and, indeed, over much of the ten records the fit is good; occasions where this is less so are discussed below in the context of the processes involved.

The specific yield value plays a major role in determining the *amplitude* of annual oscillation, given a particular input; the form of the transfer function affects the

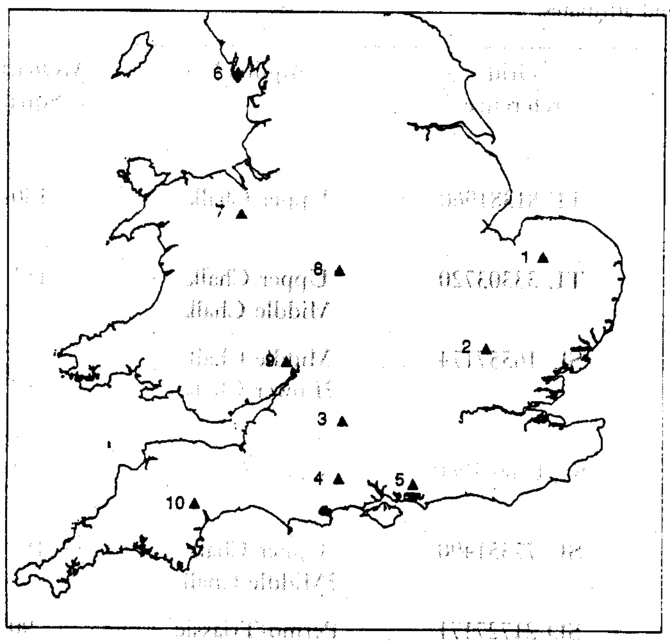


Fig. 1 *Locations of borehole sites: 1–5 chalk, 6–10 Permo-Triassic sandstones.*

*shape* of the (irregular) oscillation. What may be less immediately apparent is the degree of sensitivity of results to the parameters of the modelling. The specific yield value is of comparatively great importance, hence the suggestion that it is derived by calibration if possible rather than a value used from literature sources: a local test pumping may also be acceptable. The response function parameters are less of a cause of sensitivity in water level results. In a sense, then, characterisation of the response function has elements of ill-conditioning. This may, however, have to be accepted as a difficulty and handled in as appropriate a manner as possible if an answer needs to be sought.

The specific times for chalk sites where the method produces less good results are as follows. At sites 1 and 2 in the east of England, the observed water levels fall below the modelled in 1973, 1976 and 1990–91, whereas at other times the match is good, as it is for the other three chalk sites during these periods. These were times of comparative drought in eastern England. It is interesting to note that post-1973 and post-1976 recoveries are adequately modelled (data were not available post-1991). Where falls in water level are underpredicted, it is not known whether the MORECS estimations, the linearity of the unsaturated zone transfer, the hydraulic properties of the aquifer at depth, or abstraction influences are the cause(s). At chalk site 3 in central southern England, in contrast, the modelled water level falls below the observed in 1976 and again near the end of the record. This is explained by the physical limit imposed by the reported silting of the base of the bore (British Geological Survey well record), whilst nearby aquifer water levels can be lower. Chalk sites 4 and 5 are well-modelled throughout the whole period.

Turning to the periods of less good model fit for Permo-Triassic sandstone sites, it is site 8, north of Birmingham, which attracts attention. It is suggested that water resource demands are here impinging heavily on the natural climatic fluctuation: this, from more widespread data inspection, is a feature of a number of midland sites over the modelled period. Abstraction changes can in principle be accommodated within the method by increasing the outflow term on the basis of a rough hydraulic calculation: this has not been invoked here. With regard to the other four sandstone sites of Fig. 3, the minor irregularities in matching observed levels are small in absolute terms and it is perhaps unrealistic to look for better matching with this level of detail of modelling.

The specific yield values, derived by optimisation and shown on Figs. 2 and 3, are compatible with reported regional ranges (Allen *et al.*, in press). Such reported ranges are, however, large and a site-specific value is recommended for this method.

The transfer function parameters (Figs. 2 and 3) suggest there are not mutually exclusive classes for the two major UK aquifers, nor are moderate differences in thicknesses of unsaturated zone detectable in the parameter values when an aquifer is considered on a wide areal basis. It is notable, though, that site 2 with a comparatively late response emphasis is the site with the thickest unsaturated zone (mean value 1961–91 of 73 m).

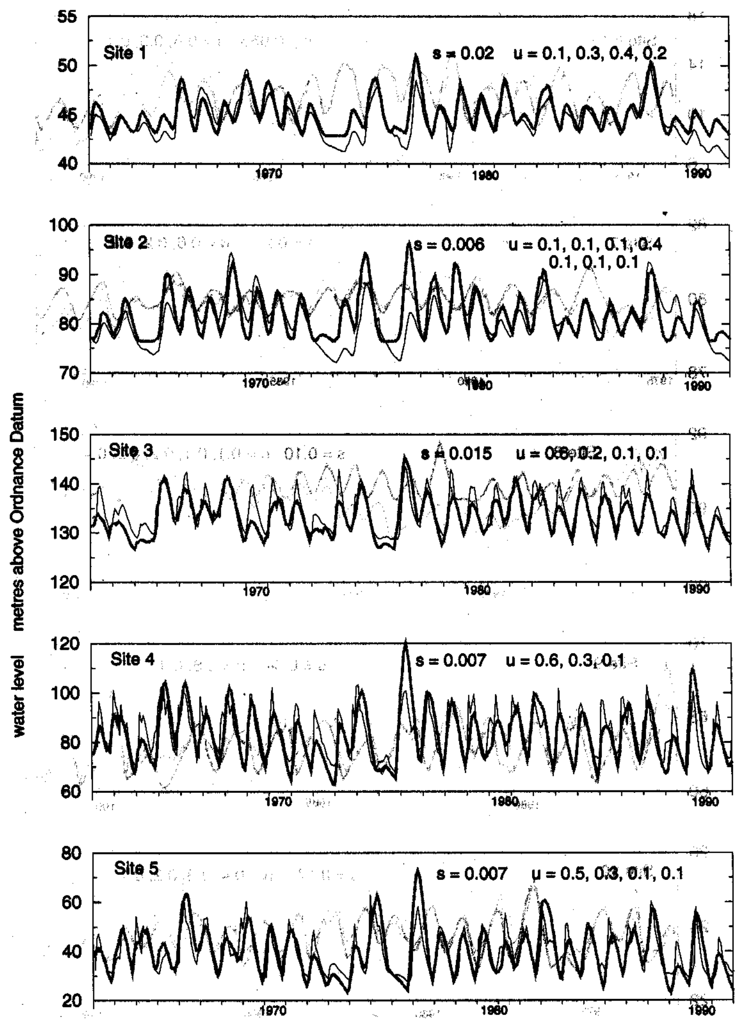


Fig. 2 *Observed and modelled water levels for the chalk sites; s is specific yield; u is the unit response function array in monthly values* ——— *observed* ——— *modelled*

In the same way as the unit hydrograph offers some information on catchment runoff processes, so the unsaturated zone transfer function gives a précis of the percolation response, similarly covering water and pulse phenomena. For seven of the ten sites, the most significant individual monthly component of saturated zone response occurs within a month of effective rainfall input. In numerical terms all the responses can be catered for within a seven-month period, but it is important to note that this may reflect an aspect of the insensitivity of the method, and it is emphasized that this is *not* necessarily to say that the water itself is all transferred to the regional saturated zone within that period.

For the chalk of the Rhee and Cam catchments to the southwest of Cambridge, Oakes (1981) derived, by back calculation, some response functions on an annual event basis: these provide a rare example in the literature of *calculated* functions. For four sites, infiltration lags of up to three to four months were derived, with the major component in the first month for three of the sites (and in the second month for the fourth site) for unsaturated zones of 19–43 metres in thickness. A fifth site of 70 m unsaturated zone gave up to seven months lag, with the fourth month providing the major single component. This agrees well with the result from site 2 of this paper. Oakes recommended no lag for the chalk of this area for unsaturated zones of less than 15 m.

For the Permo-Triassic Sherwood Sandstone of Nottinghamshire, Bishop and Rushton (1993) offered infiltration lags 'established by comparing the response of rest water levels, at observation wells, to the timing of estimated precipitation recharge events'. Their estimates show all transfer within a month for unsaturated zones of less than 10 m. For greater unsaturated zone thicknesses, the length of the response was estimated to increase and the major component to be later; for 10–30 m zone depth, the length is three months and the major contribution is in the second month. The Permo-Triassic

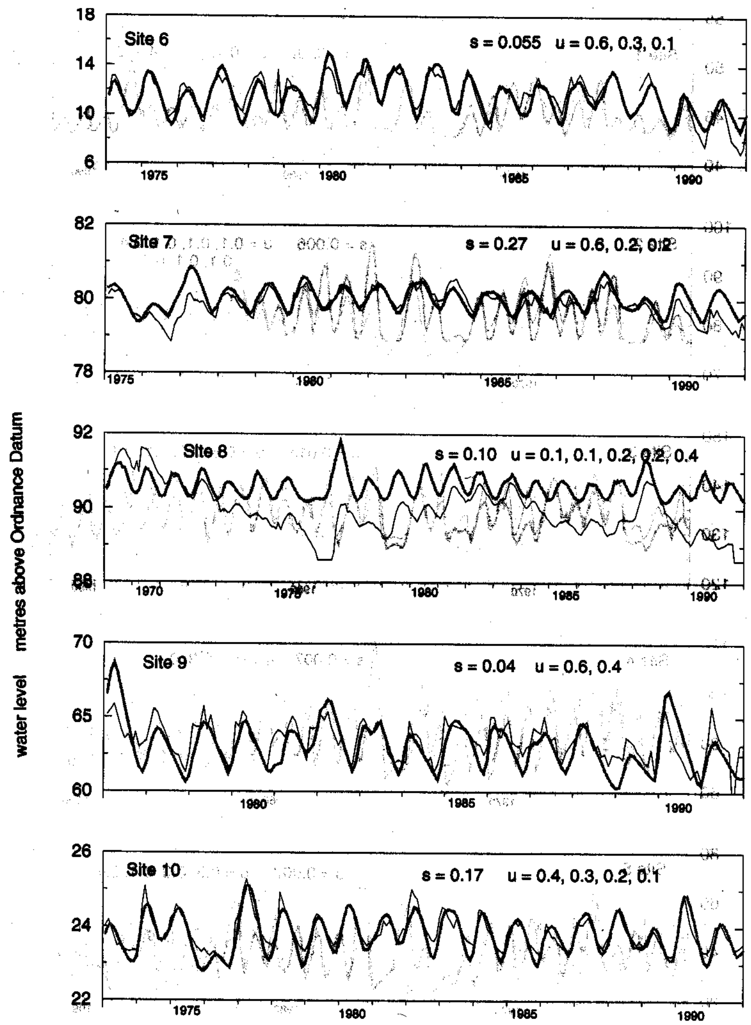


Fig. 3 *Observed and modelled water levels for the sandstone sites; s is specific yield; u is the unit response function array in monthly values*

——— *observed* ═══ *modelled*

results of the current paper are roughly compatible with these estimates but do not cover sites with average unsaturated depths greater than 10 m.

# Conclusions

1. Simple unsaturated zone transfer functions applied to effective precipitation series have been seen to give acceptable modelled water level series at a variety of topographic locations under southern UK climates for the two major UK aquifers for the period 1961–1991. These have been modelled on a continuous time basis with a simple representation of saturated zone behaviour. This representation is such that it can be used in predictive mode, without advance knowledge of water levels.
2. Strong (or intermittent) influences of water abstractions have not been specifically modelled here, but they can in principle be included in the method if required. Primarily, the method addresses variations in climatic input and any background relatively constant abstractions. It offers the possibility of simple modelling of naturalised water level series without the influence of strong abstraction regimes.
3. Typically transfer functions cover durations of three to five months, with the major single monthly component in the first month: it is stressed that this does not necessarily reflect the transfer of the water itself. The functions offer indications of hydrological process but do not replace detailed study where such is required.
4. The country-wide spread of the ten sites did not suggest mutually exclusive characterization of transfer function by aquifer type nor, within an aquifer type,

by the unsaturated zone depth, except in the more extreme case of a very extensive unsaturated zone. Transfer function results were compatible with the few reported examples. With care, some transposition for *a priori* specification for a site may become possible. Details of site hydraulics would further this aim.

5. The chief uses of the method are in water resource applications where a level of physical approximation is appropriate. Established recharge response functions offer the advantage of providing *independent* input to regional groundwater modelling studies.

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